

MODIFIED BESSEL FUNCTIONS—ORDERS 10, 11, 20 AND 21

Table 9.10

x	$10^9 x^{-10} I_{10}(x)$	$10^{11} x^{-11} I_{11}(x)$	$10^{-8} x^{10} K_{10}(x)$	$10^{24} x^{-20} I_{20}(x)$	$10^{26} x^{-21} I_{21}(x)$	$10^{-22} x^{20} K_{20}(x)$
0.0	0.26911 445	1.22324 748	1.85794 560	0.391990	0.933311	6.37771
0.2	0.26935 920	1.22426 724	1.85588 251	0.392177	0.933736	6.37435
0.4	0.27009 468	1.22733 125	1.84970 867	0.392738	0.935008	6.36429
0.6	0.27132 457	1.23245 366	1.83947 021	0.393674	0.937136	6.34757
0.8	0.27305 504	1.23965 820	1.82524 326	0.394988	0.940123	6.32424
1.0	0.27529 480	1.24897 831	1.80713 290	0.396684	0.943974	6.29437
1.2	0.27805 517	1.26045 740	1.78527 169	0.398766	0.948703	6.25807
1.4	0.28135 012	1.27414 918	1.75981 781	0.401239	0.954321	6.21545
1.6	0.28519 648	1.29011 798	1.73095 297	0.404112	0.960843	6.16665
1.8	0.28961 396	1.30843 932	1.69887 992	0.407392	0.968285	6.11184
2.0	0.29462 538	1.32920 036	1.66381 982	0.411087	0.976669	6.05118
2.2	0.30025 682	1.35250 061	1.62600 944	0.415209	0.986016	5.98488
2.4	0.30653 784	1.37845 262	1.58569 822	0.419768	0.996351	5.91314
2.6	0.31350 170	1.40718 285	1.54314 529	0.424778	1.007703	5.83620
2.8	0.32118 565	1.43883 260	1.49861 645	0.430253	1.020101	5.75428
3.0	0.32963 121	1.47355 907	1.45238 126	0.436209	1.033581	5.66764
3.2	0.33888 455	1.51153 657	1.40471 020	0.442662	1.048178	5.57655
3.4	0.34899 681	1.55295 782	1.35587 192	0.449632	1.063935	5.48128
3.6	0.36002 459	1.59803 551	1.30613 075	0.457139	1.080893	5.38210
3.8	0.37203 039	1.64700 388	1.25574 432	0.465205	1.099102	5.27932
4.0	0.38508 316	1.70012 064	1.20496 150	0.473853	1.118613	5.17321
4.2	0.39925 889	1.75766 896	1.15402 052	0.483111	1.139481	5.06408
4.4	0.41464 125	1.81995 978	1.10314 736	0.493006	1.161768	4.95224
4.6	0.43132 237	1.88733 435	1.05255 442	0.503569	1.185538	4.83797
4.8	0.44940 362	1.96016 700	1.00243 944	0.514832	1.210861	4.72159
5.0	0.46899 655	2.03886 82	0.95298 465	0.526830	1.237813	4.60339
5.2	0.49022 387	2.12388 83	0.90435 626	0.539601	1.266475	4.48367
5.4	0.51322 061	2.21572 08	0.85670 405	0.553186	1.296933	4.36272
5.6	0.53813 536	2.31490 71	0.81016 129	0.567630	1.329281	4.24084
5.8	0.56513 169	2.42204 09	0.76484 483	0.582979	1.363622	4.11830
6.0	0.59438 965	2.53777 36	0.72085 532	0.599284	1.400061	3.99537
6.2	0.62610 759	2.66282 00	0.67827 767	0.616599	1.438715	3.87234
6.4	0.66050 400	2.79796 48	0.63718 161	0.634984	1.479709	3.74945
6.6	0.69781 972	2.94406 93	0.59762 235	0.654501	1.523176	3.62695
6.8	0.73832 033	3.10208 00	0.55964 137	0.675219	1.569259	3.50507
7.0	0.78229 881	3.27303 69	0.52326 729	0.697210	1.618113	3.38405
7.2	0.83007 854	3.45808 34	0.48851 672	0.720554	1.669904	3.26411
7.4	0.88201 663	3.65847 74	0.45539 529	0.745333	1.724808	3.14543
7.6	0.93850 764	3.87560 29	0.42389 854	0.771639	1.783016	3.02821
7.8	0.99998 773	4.11098 38	0.39401 295	0.799570	1.844734	2.91264
8.0	1.06693 936	4.36629 90	0.36571 690	0.829231	1.910180	2.79887
8.2	1.13989 641	4.64339 88	0.33898 159	0.860735	1.979593	2.68705
8.4	1.21945 007	4.94432 35	0.31377 202	0.894204	2.053225	2.57733
8.6	1.30625 534	5.27132 42	0.29004 783	0.929769	2.131351	2.46983
8.8	1.40103 829	5.62688 64	0.26776 418	0.967571	2.214264	2.36466
9.0	1.50460 429	6.01375 48	0.24687 251	1.007764	2.302281	2.26193
9.2	1.61784 713	6.43496 31	0.22732 134	1.050510	2.395741	2.16172
9.4	1.74175 933	6.89386 57	0.20905 690	1.095988	2.495011	2.06411
9.6	1.87744 369	7.39417 36	0.19202 382	1.144389	2.600488	1.96916
9.8	2.02612 620	7.93999 51	0.17616 568	1.195919	2.712593	1.87692
10.0	2.18917 062	8.53588 02	0.16142 553	1.250800	2.831786	1.78744
	$\begin{bmatrix} (-3)2 \\ 7 \end{bmatrix}$	$\begin{bmatrix} (-3)6 \\ 6 \end{bmatrix}$	$\begin{bmatrix} (-4)5 \\ 6 \end{bmatrix}$	$\begin{bmatrix} (-4)4 \\ 4 \end{bmatrix}$	$\begin{bmatrix} (-4)9 \\ 5 \end{bmatrix}$	$\begin{bmatrix} (-4)8 \\ 4 \end{bmatrix}$

$$I_{n+1}(x) = -\frac{2n}{x} I_n(x) + I_{n-1}(x)$$

$$K_{n+1}(x) = \frac{2n}{x} K_n(x) + K_{n-1}(x)$$

Compiled from British Association for the Advancement of Science, Bessel functions, Part II. Functions of positive integer order, Mathematical Tables, vol. X (Cambridge Univ. Press, Cambridge, England, 1952) and L. Fox, A short table for Bessel functions of integer orders and large arguments. Royal Society Shorter Mathematical Tables No. 3 (Cambridge Univ. Press, Cambridge, England, 1954) (with permission).

Table 9.10 MODIFIED BESSEL FUNCTIONS—ORDERS 10, 11, 20 AND 21

x	$e^{-x}I_{10}(x)$	$e^{-x}I_{11}(x)$	$e^xK_{10}(x)$	$10^{24}x^{-20}I_{20}(x)$	$10^{26}x^{-21}I_{21}(x)$	$10^{-22}x^{20}K_{20}(x)$
10.0	0.00099 38819	0.00038 75284	35.55633 91	1.25080	2.83179	1.787443
10.2	0.00107 29935	0.00042 45861	32.60759 68	1.30927	2.95856	1.700753
10.4	0.00115 52835	0.00046 37417	29.98423 91	1.37160	3.09345	1.616873
10.6	0.00124 06973	0.00050 50080	27.64297 29	1.43806	3.23703	1.535814
10.8	0.00132 91744	0.00054 83934	25.54714 23	1.50895	3.38992	1.457578
11.0	0.00142 06490	0.00059 39013	23.66558 79	1.58462	3.55278	1.382160
11.2	0.00151 50508	0.00064 15309	21.97172 20	1.66540	3.72634	1.309546
11.4	0.00161 23051	0.00069 12768	20.44277 46	1.75169	3.91139	1.239714
11.6	0.00171 23339	0.00074 31298	19.05917 72	1.84390	4.10876	1.172637
11.8	0.00181 50559	0.00079 70766	17.80405 56	1.94249	4.31937	1.108279
12.0	0.00192 03870	0.00085 31003	16.66281 24	2.04795	4.54421	1.046601
12.2	0.00202 82412	0.00091 11805	15.62277 97	2.16080	4.78434	0.987556
12.4	0.00213 85303	0.00097 12937	14.67293 16	2.28162	5.04093	0.931095
12.6	0.00225 11650	0.00103 34132	13.80364 34	2.41105	5.31521	0.877164
12.8	0.00236 60548	0.00109 75097	13.00649 01	2.54975	5.60856	0.825703
13.0	0.00248 31086	0.00116 35512	12.27407 71	2.69846	5.92244	0.776652
13.2	0.00260 22347	0.00123 15035	11.59989 74	2.85799	6.25845	0.729947
13.4	0.00272 33415	0.00130 13301	10.97821 07	3.02921	6.61832	0.685520
13.6	0.00284 63375	0.00137 29926	10.40394 07	3.21306	7.00393	0.643305
13.8	0.00297 11314	0.00144 64509	9.87258 79	3.41058	7.41731	0.603230
14.0	0.00309 76327	0.00152 16634	9.38015 52	3.62289	7.86068	0.565225
14.2	0.00322 57518	0.00159 85870	8.92308 36	3.85121	8.33644	0.529218
14.4	0.00335 53999	0.00167 71776	8.49819 79	4.09686	8.84722	0.495137
14.6	0.00348 64894	0.00175 73898	8.10265 95	4.36131	9.39585	0.462910
14.8	0.00361 89341	0.00183 91776	7.73392 53	4.64613	9.98543	0.432464
15.0	0.00375 26491	0.00192 24942	7.38971 31	4.95305	10.61932	0.403728
15.2	0.00388 75510	0.00200 72921	7.06797 04	5.28394	11.30119	0.376630
15.4	0.00402 35583	0.00209 35235	6.76684 87	5.64087	12.03503	0.351101
15.6	0.00416 05908	0.00218 11403	6.48467 94	6.02608	12.82520	0.327070
15.8	0.00429 85705	0.00227 00942	6.21995 46	6.44202	13.67643	0.304470
16.0	0.00443 74209	0.00236 03366	5.97130 87	6.89137	14.59389	0.283235
16.2	0.00457 70675	0.00245 18192	5.73750 35	7.37705	15.58322	0.263299
16.4	0.00471 74378	0.00254 44936	5.51741 43	7.90228	16.65059	0.244598
16.6	0.00485 84612	0.00263 83118	5.31001 78	8.47055	17.80271	0.227071
16.8	0.00500 00690	0.00273 32259	5.11438 19	9.08571	19.04691	0.210658
17.0	0.00514 21947	0.00282 91884	4.92965 63	9.75197	20.39124	0.195301
17.2	0.00528 47735	0.00292 61523	4.75506 40	10.47392	21.84444	0.180944
17.4	0.00542 77427	0.00302 40709	4.58989 42	11.25663	23.41611	0.167532
17.6	0.00557 10418	0.00312 28982	4.43349 60	12.10562	25.11674	0.155012
17.8	0.00571 46119	0.00322 25887	4.28527 20	13.02697	26.95781	0.143336
18.0	0.00585 83964	0.00332 30977	4.14467 40	14.02734	28.95188	0.132454
18.2	0.00600 23403	0.00342 43808	4.01119 75	15.11406	31.11272	0.122321
18.4	0.00614 63909	0.00352 63948	3.88437 85	16.29515	33.45541	0.112891
18.6	0.00629 04971	0.00362 90969	3.76378 89	17.57946	35.99648	0.104124
18.8	0.00643 46098	0.00373 24450	3.64903 41	18.97668	38.75407	0.095978
19.0	0.00657 86817	0.00383 63982	3.53974 93	20.49749	41.74804	0.088414
19.2	0.00672 26672	0.00394 09161	3.43559 74	22.15363	45.00024	0.081397
19.4	0.00686 65226	0.00404 59590	3.33626 62	23.95803	48.53460	0.074892
19.6	0.00701 02059	0.00415 14885	3.24146 65	25.92489	52.37745	0.068865
19.8	0.00715 36768	0.00425 74667	3.15093 00	28.06989	56.55768	0.063285
20.0	0.00729 68965 $\left[\begin{smallmatrix} (-7)4 \\ 5 \end{smallmatrix} \right]$	0.00436 38567 $\left[\begin{smallmatrix} (-7)3 \\ 5 \end{smallmatrix} \right]$	3.06440 75 $\left[\begin{smallmatrix} (-2)4 \\ 8 \end{smallmatrix} \right]$	30.41029 $\left[\begin{smallmatrix} (-2)2 \\ 5 \end{smallmatrix} \right]$	61.10706 $\left[\begin{smallmatrix} (-2)5 \\ 5 \end{smallmatrix} \right]$	0.058124 $\left[\begin{smallmatrix} (-4)4 \\ 4 \end{smallmatrix} \right]$

MODIFIED BESSEL FUNCTIONS—AUXILIARY TABLE FOR LARGE ARGUMENTS

Table 9.10

x^{-1}	$\ln [x^{\frac{1}{2}}e^{-x}I_{10}(x)]$	$\ln [x^{\frac{1}{2}}e^{-x}I_{11}(x)]$	$\ln [\pi^{-\frac{1}{2}}x^{\frac{1}{2}}e^{-x}K_{10}(x)]$	$\ln [x^{\frac{1}{2}}e^{-x}I_{20}(x)]$	$\ln [x^{\frac{1}{2}}e^{-x}I_{21}(x)]$	$\ln [\pi^{-\frac{1}{2}}x^{\frac{1}{2}}e^{-x}K_{20}(x)]$	$\langle x \rangle$
0.050	-3.42244 002	-3.93653 292	1.47299 048	-10.434749	-11.346341	8.250182	20
0.049	-3.37318 689	-3.87762 888	1.42771 939	-10.263511	-11.160467	8.088946	20
0.048	-3.32386 306	-3.81861 524	1.38232 785	-10.091302	-10.973471	7.926737	21
0.047	-3.27447 055	-3.75949 454	1.33681 644	-9.918126	-10.785351	7.763551	21
0.046	-3.22501 139	-3.70026 938	1.29118 575	-9.743983	-10.596108	7.599386	22
0.045	-3.17548 766	-3.64094 242	1.24543 642	-9.568876	-10.405744	7.434240	22
0.044	-3.12590 147	-3.58151 639	1.19956 910	-9.392809	-10.214259	7.268110	23
0.043	-3.07625 496	-3.52199 408	1.15358 449	-9.215785	-10.021658	7.100994	23
0.042	-3.02655 033	-3.46237 835	1.10748 332	-9.037810	-9.827944	6.932893	24
0.041	-2.97678 979	-3.40267 211	1.06126 635	-8.858889	-9.633121	6.763806	24
0.040	-2.92697 559	-3.34287 833	1.01493 437	-8.679029	-9.437195	6.593733	25
0.039	-2.87711 002	-3.28300 006	0.96848 822	-8.498236	-9.240173	6.422673	26
0.038	-2.82719 539	-3.22304 039	0.92192 874	-8.316519	-9.042063	6.250630	26
0.037	-2.77723 405	-3.16300 246	0.87525 686	-8.133888	-8.842873	6.077603	27
0.036	-2.72722 837	-3.10288 949	0.82847 349	-7.950352	-8.642612	5.903597	28
0.035	-2.67718 076	-3.04270 472	0.78157 961	-7.765923	-8.441293	5.728614	29
0.034	-2.62709 365	-2.98245 146	0.73457 624	-7.580613	-8.238927	5.552659	29
0.033	-2.57696 948	-2.92213 308	0.68746 441	-7.394434	-8.035529	5.375732	30
0.032	-2.52681 074	-2.86175 298	0.64024 520	-7.207403	-7.831113	5.197843	31
0.031	-2.47661 992	-2.80131 461	0.59291 975	-7.019533	-7.625695	5.018998	32
0.030	-2.42639 955	-2.74082 147	0.54548 920	-6.830842	-7.419294	4.839203	33
0.029	-2.37615 216	-2.68027 709	0.49795 475	-6.641348	-7.211929	4.658466	34
0.028	-2.32588 032	-2.61968 504	0.45031 764	-6.451070	-7.003620	4.476796	36
0.027	-2.27558 659	-2.55904 894	0.40257 915	-6.260027	-6.794389	4.294202	37
0.026	-2.22527 356	-2.49837 243	0.35474 059	-6.068243	-6.584261	4.110696	38
0.025	-2.17494 384	-2.43765 918	0.30680 331	-5.875738	-6.373261	3.926290	40
0.024	-2.12460 002	-2.37691 291	0.25876 871	-5.682539	-6.161416	3.740995	42
0.023	-2.07424 475	-2.31613 733	0.21063 822	-5.488669	-5.948754	3.554826	43
0.022	-2.02388 063	-2.25533 620	0.16241 332	-5.294155	-5.735305	3.367799	45
0.021	-1.97351 031	-2.19451 329	0.11409 551	-5.099025	-5.521102	3.179929	48
0.020	-1.92313 643	-2.13367 239	0.06568 636	-4.903309	-5.306177	2.991233	50
0.019	-1.87276 162	-2.07281 731	+0.01718 745	-4.707035	-5.090565	2.801730	53
0.018	-1.82238 853	-2.01195 186	-0.03139 959	-4.510235	-4.874302	2.611440	56
0.017	-1.77201 979	-1.95107 986	-0.08007 306	-4.312943	-4.657427	2.420383	59
0.016	-1.72165 806	-1.89020 514	-0.12883 128	-4.115190	-4.439978	2.228582	63
0.015	-1.67130 595	-1.82933 153	-0.17767 247	-3.917011	-4.221995	2.036059	67
0.014	-1.62096 610	-1.76846 286	-0.22659 485	-3.718443	-4.003521	1.842840	71
0.013	-1.57064 113	-1.70760 295	-0.27559 659	-3.519520	-3.784599	1.648949	77
0.012	-1.52033 365	-1.64675 564	-0.32467 581	-3.320281	-3.565272	1.454415	83
0.011	-1.47004 626	-1.58592 472	-0.37383 061	-3.120763	-3.345586	1.259264	91
0.010	-1.41978 154	-1.52511 400	-0.42305 904	-2.921004	-3.125587	1.063526	100
0.009	-1.36954 207	-1.46432 725	-0.47235 911	-2.721043	-2.905322	0.867231	111
0.008	-1.31933 040	-1.40356 824	-0.52172 881	-2.520921	-2.684838	0.670412	125
0.007	-1.26914 908	-1.34284 072	-0.57116 608	-2.320676	-2.464184	0.473099	143
0.006	-1.21900 063	-1.28214 841	-0.62066 881	-2.120350	-2.243408	0.275328	167
0.005	-1.16888 754	-1.22149 499	-0.67023 489	-1.919982	-2.022558	+0.077133	200
0.004	-1.11881 229	-1.16088 414	-0.71986 215	-1.719613	-1.801685	-0.121451	250
0.003	-1.06877 735	-1.10031 949	-0.76954 839	-1.519284	-1.580838	-0.320388	333
0.002	-1.01878 514	-1.03980 463	-0.81929 138	-1.319036	-1.360065	-0.519640	500
0.001	-0.96883 808	-0.97934 314	-0.86908 886	-1.118907	-1.139416	-0.719170	1000
0.000	-0.91893 853	-0.91893 853	-0.91893 853	-0.918939	-0.918939	-0.918939	∞
	$\begin{bmatrix} (-6)9 \\ 4 \end{bmatrix}$	$\begin{bmatrix} (-5)1 \\ 4 \end{bmatrix}$	$\begin{bmatrix} (-5)2 \\ 4 \end{bmatrix}$	$\begin{bmatrix} (-4)1 \\ 4 \end{bmatrix}$	$\begin{bmatrix} (-4)1 \\ 4 \end{bmatrix}$	$\begin{bmatrix} (-4)1 \\ 4 \end{bmatrix}$	

 $\langle x \rangle$ = nearest integer to x .

Compiled from L. Fox, A short table for Bessel functions of integer orders and large arguments. Royal Society Shorter Mathematical Tables No. 3 (Cambridge Univ. Press, Cambridge, England, 1954) (with permission).